HEAT PROPAGATION IN MEDIA IN THE PRESENCE OF INTERNAL CONVERSIONS

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UDC 536.42

A phenomenological approach to describing heat propagation in media with internal conversions is considered.

With change in temperature in multicomponent media, various processes may occur, such as solution, the destruction and formation of structure, chemical reactions, etc. Taking these phenomena into account in heat transfer usually leads to equations of higher order than the classical heat-conduction equation [1-4].

These processes are characterized, close to equilibrium, by the possibility of being described by a single type of equation

$$\alpha \frac{\partial \eta}{\partial t} + \eta = F,$$

where η is some quantitative characteristic, for example, the mass of dissolved material or the structure-destruction parameter, while F is an external perturbation. A characteristic feature of such processes is that the theory of local thermodynamic equilibrium cannot be applied to them. In the present work, an approach based on a principle of nonequilibrium thermodynamics is considered; the principle in question is the principle of minimum energy scattering [5].

Suppose that when heat propagates in a medium some internal conversion occurs. Let A denote the affinity to conversion and ξ the corresponding internal variable. Then, disregarding the change in density, the Gibbs equation takes the form [6]

$$du = Tds + Ad\xi, \tag{1}$$

and hence

$$\frac{\partial s}{\partial t} = \frac{1}{T}\frac{\partial u}{\partial t} - \frac{A}{T}\frac{\partial \xi}{\partial t} = \frac{c}{T}\frac{\partial T}{\partial t} + \frac{a}{T}\frac{\partial A}{\partial t} - \frac{A}{T}\frac{\partial \xi}{\partial t}, \tag{2}$$

where $a = (\partial u/\partial A)_{TV}$; $c = (\partial u/\partial T)_{VA}$.

In deriving the equations of heat propagation, the principle of minimum energy scattering will be taken as the starting point; the scattering potential is expressed in terms of the thermodynamic forces [5]

$$\delta \int\limits_V (\sigma - \Psi)\, dV = 0, \tag{3}$$

where the variation is performed with constant fluxes, and $\sigma = \rho \frac{\partial s}{\partial t} + \nabla J_s$.

Using the entropic representation, the two forces X_1 and X_2 in the present case may be written in the form $X_1 = \nabla \frac{1}{T}$, $X_2 = \frac{A}{T}$. Then, in the linear approximation, the corresponding fluxes take the form: $J_1 = L\nabla \frac{1}{T}$, $J_2 = \varepsilon \frac{A}{T}$, and the scattering potential is

$$\Psi = \frac{L}{2}\left(\nabla \frac{1}{T}\right)^2 + \frac{\varepsilon}{2}\left(\frac{A}{T}\right)^2. \tag{4}$$

Applying the divergence theorem to Eq. (3), it is found that

Ufimskii Petroleum Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 5, pp. 829-831, May, 1984. Original article submitted March 25, 1983.

0022-0841/84/4605-0605$08.50

$$\delta \int_V \left(\rho \frac{\partial s}{\partial t} - \Psi\right) dV + \delta \oint_\Omega J_s d\Omega = 0. \tag{5}$$

Assuming that variation is not taken over the boundary surface, i.e., J_s and T are fixed with respect to the boundary surface Ω, and using Eqs. (2) and (4), Eq. (5) takes the form

$$\delta \int_V \left(\rho \frac{c}{T} \frac{\partial T}{\partial t} + \rho \frac{a}{T} \frac{\partial A}{\partial t} - \rho \frac{A}{T} \frac{\partial \xi}{\partial t} - \frac{L}{2} \left(\nabla \frac{1}{T}\right)^2 - \frac{\varepsilon}{2} \left(\frac{A}{T}\right)^2\right) dV = 0. \tag{6}$$

Since the fluxes are not varied, it follows from the energy-conservation law

$$\rho c \frac{\partial T}{\partial t} + \rho a \frac{\partial A}{\partial t} = -\nabla J_1$$

that the variation of the first two terms in Eq. (6) is equal to zero. Then Eq. (6) may be written in the form

$$\int_V \left(\left(\rho c \frac{\partial T}{\partial t} + \rho a \frac{\partial A}{\partial t} + \nabla \left(L \nabla \frac{1}{T}\right)\right) \delta \left(\frac{1}{T}\right) - \left(\varepsilon \frac{A}{T} + \rho \frac{\partial \xi}{\partial t}\right) \delta \left(\frac{A}{T}\right)\right) dV = 0. \tag{7}$$

Hence

$$\rho \frac{\partial \xi}{\partial t} = -\frac{\varepsilon}{T} A, \tag{8}$$

$$\rho c \frac{\partial T}{\partial t} + \rho a \frac{\partial A}{\partial t} = \nabla (\lambda \nabla T), \tag{9}$$

where $\lambda = L/T^2$.

Taking into account that the internal variable is a function of A and T, i.e., $\xi = \xi(A, T)$, it follows from Eq. (8) that

$$\alpha \frac{\partial A}{\partial t} + A = b \frac{\partial T}{\partial t}, \tag{10}$$

where $\alpha = \frac{\rho T}{\varepsilon} \frac{\partial \xi}{\partial A}$; $b = -\frac{\rho T}{\varepsilon} \frac{\partial \xi}{\partial T}$.

Thus, the system obtained in Eqs. (9) and (10) describes the heat propagation in media with internal conversions, taking account of the assumptions imposed on Eq. (1).

It may readily be shown that, in the case of constant coefficients, Eqs. (9) and (10) may be reduced to the equation of generalized heat conduction

$$\beta \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \varkappa \left(\alpha \frac{\partial}{\partial t} + 1\right) \Delta T, \tag{11}$$

where $\varkappa = \lambda/\rho c$ is the thermal diffusivity; $\beta = \alpha + \frac{ab}{c}$.

The specific physical interpretation of the parameters α and β may vary. Thus, in considering the heat exchange in non-Newtonian media, an equation of the type in Eq. (11) obtained by generalizing the Fourier law is used [2]. The coefficients α and β are then interpreted as relaxational parameters. It may also be shown that the system of equations used in investigating thermal phenomena in disperse media [3] may be reduced to a system of the type in Eqs. (9) and (10). The temperature of the disperse phase may then be interpreted as an internal variable, and the temperature deviation of the dispersional phase from the disperse phase as the affinity for conversion.

As shown in [4], the description of the heat transfer in nonequilibrium from the viewpoint of the relaxational formalism of the thermodynamics of irreversible processes leads to an equation of the type in Eq. (11).

Note that, for this type of equation, the method of solving inverse problems has been well developed; this is of practical interest in the solution of many engineering problems associated with heat-transfer questions.

NOTATION

T, temperature; A, affinity for conversion; ξ, internal variable; u, specific internal energy; s, specific entropy; ρ, density; J_s, entropy flux; Ψ, local scattering potential; c, specific heat; λ, thermal conductivity.

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STRONG TEMPERATURE-FIELD DISCONTINUITIES IN A NONLINEAR MEDIUM

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UDC 536.2.01

The conditions for the appearance of temperature discontinuities in a nonlinear medium with finite relaxation times of the heat flux are analyzed.

General laws of the appearance and propagation of simple and shock waves in heat-transfer theory were discussed in [1, 2]; the good prospects for using gasdynamic methods in modeling high-intensity thermal processes was noted.

Below, gasdynamic methods are used to investigate the wave equation of heat transfer and construct its solution behind the front of a strong temperature discontinuity.

1. Simple Heat Waves and the Appearance of a Strong Heat-Field Discontinuity

The generalized heat-transfer equations of [2, 3] form the starting point

$$cT_t + c\gamma T_{tt} = (\lambda T_x)_x, \tag{1}$$

$$q = -\lambda T_x - \gamma q_t. \tag{2}$$

If high-intensity nonsteady heat transfer occurs [1], or the thermophysical properties of the medium are such that the influence of the thermal-relaxation parameter γ is significant [4], then the quantity cT_t in Eq. (1) may be neglected. Adopting this assumption and setting $\lambda = \lambda(T)$, c, γ = const, it is found that

$$T_{tt} = \left(\frac{\lambda}{c\gamma} T_x\right)_x. \tag{3}$$

Obviously, Eq. (3) is equivalent to a system of quasilinear equations

$$T_t = R_x,\ R_t = L_x,\ L = L(T);\ \ \frac{dL(T)}{dT} \equiv \dot{L}(T) = \frac{\lambda}{c\gamma} > 0. \tag{4}$$

Gomel'skii Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 5, pp. 832-836, May, 1984. Original article submitted November 18, 1982.